

Basic beliefs and argument-based beliefs in awareness epistemic logic with structured arguments^a

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Motivation

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P2 Argument evaluation is conditioned by belief formation.

Premises that are believed should be preferred by the agent to premisses that are not believed.

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- P2 in **formal argumentation**: ordering premisses according to their reliability (Beirlaen et al., 2018).
- P2 in **epistemic justification logic**: acceptance of a complex piece of evidence (Baltag et al., 2012)= belief of its basic parts (premisses).

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\mathcal{L}_{BA} and its semantics

Basic Beliefs and AB-Beliefs

Ongoing and future work

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$$\alpha ::= \langle \varphi \rangle \mid \langle \alpha_1, \dots, \alpha_n \rightarrow \varphi \rangle \mid \langle \alpha_1, \dots, \alpha_n \Rightarrow \varphi \rangle \quad \varphi \in \mathcal{L}_{\text{BA}}$$

Single negation

$$\sim \varphi := \begin{cases} \psi & \text{if } \varphi \text{ is of the form } \neg\psi; \\ \neg\varphi & \text{otherwise.} \end{cases}$$

Argument Structure (Modgil and Prakken, 2014)

- $\text{Prem}(\alpha)$ returns the *premisses* of α **Example:**
 $\text{Prem}(\langle\langle\langle\langle p \rangle, \langle q \rangle \Rightarrow r \rangle \Rightarrow s \rangle \rightarrow s \vee t \rangle) = \{p, q\}$.
- $\text{Conc}(\alpha)$ returns the *conclusion* of α **Example:**
 $\text{Conc}(\langle\langle\langle\langle p \rangle, \langle q \rangle \Rightarrow r \rangle \Rightarrow s \rangle \rightarrow s \vee t \rangle) = s \vee t$.
- $\text{sub}_A(\alpha)$ returns the *subarguments* of α .
- $\text{TopRule}(\alpha)$ returns the *top rule* of α **Example:**
 $\text{TopRule}(\langle\langle\langle\langle p \rangle, \langle q \rangle \Rightarrow r \rangle \Rightarrow s \rangle \rightarrow s \vee t \rangle) = (s, s \vee t)$.
- $\text{DefRule}(\alpha)$ returns the set of *defeasible rules* of α
Example: $\text{DefRule}(\langle\langle\langle\langle p \rangle, \langle q \rangle \Rightarrow r \rangle \Rightarrow s \rangle \rightarrow s \vee t \rangle) = \{((p, q), r), ((r), s)\}$.

\mathcal{L}_{BA} -models and Truth i

A **model** for \mathcal{L}_{BA} is a tuple $M = (W, \mathcal{B}, \mathcal{O}, \mathcal{D}, n, \|\cdot\|)$ where:

- $W \neq \emptyset$ (*possible worlds*)
- $\mathcal{B} \subseteq W$ and $\mathcal{B} \neq \emptyset$ (*doxastically indistinguishably worlds*)
 - $M, w \models \Box\varphi$ iff $w' \in \mathcal{B}$ implies $M, w' \models \varphi$
- $\mathcal{O} \subseteq_{\text{fin}} Ar$ (*awareness set of the agent*)
 - $M, w \models \text{aware}(\alpha)$ iff $\alpha \in \mathcal{O}$
- $\mathcal{D} \subseteq_{\text{fin}} \mathcal{L}_{BA}^n \times \mathcal{L}_{BA}$ (with $n \in \mathbb{N}$) (*accepted defeasible rules*).
 - $M, w \models \text{wellshap}(\langle\varphi\rangle)$
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 $M, w \models \text{wellshap}(\alpha_i)$ for every $1 \leq i \leq n$ and
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 $((\text{Conc}(\alpha_1), \dots, \text{Conc}(\alpha_n)), \varphi) \in \mathcal{D}$
- $\mathfrak{n} : \mathcal{D} \rightarrow \mathbb{P}$ ((possibly partial) *naming function*), where $\mathfrak{n}(R)$ means “the defeasible rule R is applicable”.
 - $M, w \models \text{undercuts}(\alpha, \beta)$ iff $\text{Conc}(\alpha) = \sim r$ and $\mathfrak{n}(\text{TopRule}(\beta)) = r$
 - $M, w \models \text{conc}(\alpha) = \varphi$ iff $\text{Conc}(\alpha) = \varphi$
 - $M, w \models \text{strict}(\alpha)$ iff $\text{DefRule}(\alpha) = \emptyset$

Basic Beliefs and AB-Beliefs

Basic beliefs

(basic implicit belief) $\Box\varphi$

(basic explicit belief) $\Box^e\varphi := \Box\varphi \wedge \text{aware}(\langle\varphi\rangle)$

- “Positive” informal reading (reasonable assumptions, sound observations,...)
- “Negative” informal reading (biases, prejudices,...).

Attacks

- (undermining: attacking ordinary premisses)

$$\text{undermines}(\alpha, \beta) := \bigvee_{\varphi \in \text{Prem}(\beta)} \text{conc}(\alpha) =_{\sim} \varphi$$

- (rebuttal: attacking the conclusion)

$$\text{rebuts}(\alpha, \beta) := \bigvee_{\langle \beta_1, \dots, \beta_n \hookrightarrow \varphi \rangle \in \text{sub}_A(\beta)} \text{conc}(\alpha) =_{\sim} \varphi$$

where $\hookrightarrow \in \{ \rightarrow, \Rightarrow \}$

- (undercutting: attacking the inference link)

undercuts is a primitive operator in \mathcal{L}_{BA} :

$$M, w \models \text{undercuts}(\alpha, \beta) \text{ iff } \text{Conc}(\alpha) =_{\sim} r \text{ and } \mathfrak{n}(\text{TopRule}(\beta)) = r$$

Attacks

- (undercutting*: attacking a defeasible inference link of some subargument)

$$\text{undercuts}^*(\alpha, \beta) = \bigvee_{\beta' \in \text{sub}_A(\beta)} \text{undercuts}(\alpha, \beta')$$

We are finally able to express what does it mean for an argument to attack another:

$$\text{attacks}(\alpha, \beta) := \text{undermines}(\alpha, \beta) \vee \text{rebutts}(\alpha, \beta) \vee \text{undercuts}^*(\alpha, \beta)$$

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Defeat and Associated AF

Defeat=successful attack, similar to ASPIC⁺, except for:

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Given (M, w) , where $M = (W, \mathcal{B}, \mathcal{O}, \mathcal{D}, \mathfrak{n}, \|\cdot\|)$. The **AF associated to** (M, w) is $AF^M = (\mathcal{O}^{\text{ws}}, \rightsquigarrow)$ where

$\mathcal{O}^{\text{ws}} := \{\alpha \in \mathcal{O} \mid M, w \models \text{wellshap}(\alpha)\}$ and $\rightsquigarrow \subseteq \mathcal{O}^{\text{ws}} \times \mathcal{O}^{\text{ws}}$ is defined as $\alpha \rightsquigarrow \beta$ iff $M, w \models \text{defeat}(\alpha, \beta)$

AB-Beliefs

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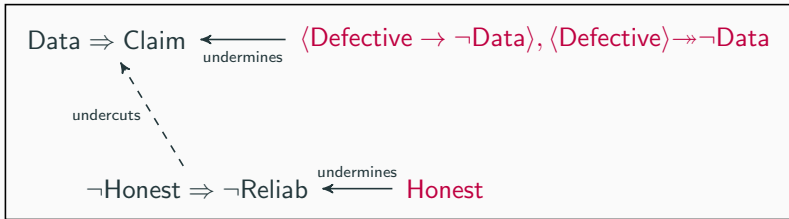
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- B^{AB} cannot be captured in \mathcal{L}_{BA} , therefore we need to enrich \mathcal{L}_{BA} with B^{AB} .

An example



AF^M

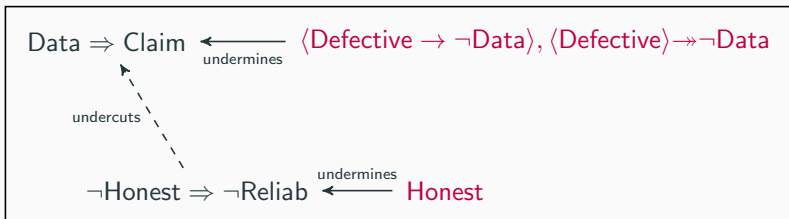


$GE(AF^M)$

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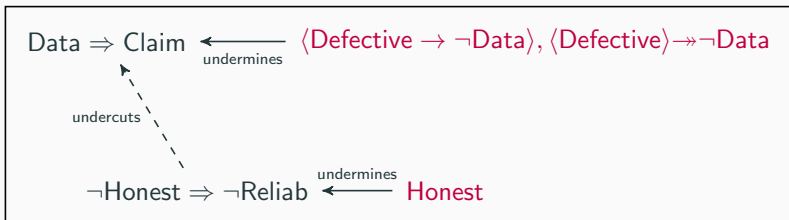
$GE(AF^M)$

$M, w \models \Box^e \text{Honest}$

An example



AF^M



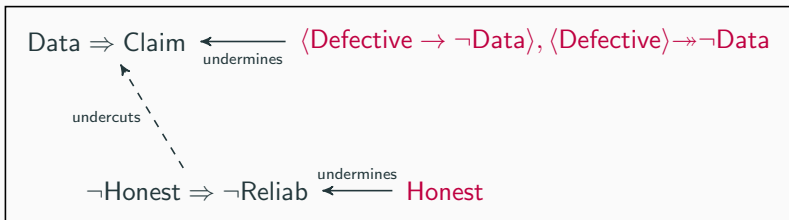
$GE(AF^M)$

$M, w \models \neg B^{AB} \text{Claim}$

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AF^M



$GE(AF^M)$

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Ongoing and future work

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Thank you!

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