An Adjustment Function For Dealing With Similarities

Leila Amgoud Victor David

IRIT, CNRS, Université de Toulouse
31062 Toulouse Cedex 9, France

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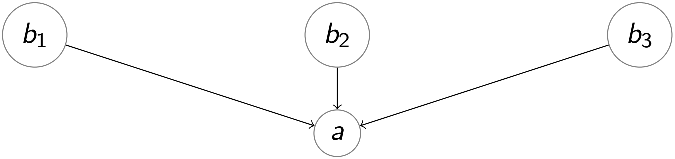
Introduction

Argumentation is a reasoning approach based on interacting arguments

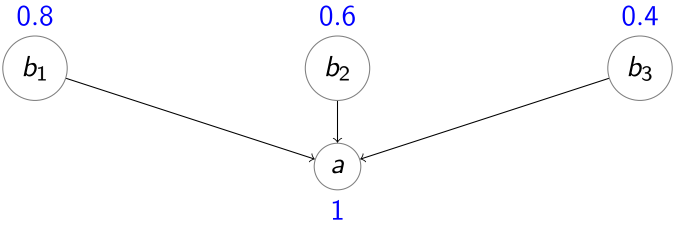
[a] My salary is quite high then I can rent the apartment.
[b_1] If the bank approves the loan to buy a car, I will not have enough money to rent the apartment.
[b_2] I may adopt a dog, however the owner forbids pets, thus I will not rent the apartment.
[b_3] I may adopt a cat, however the owner forbids pets, thus I will not rent the apartment.



Weighted Arguments

For instance: certainty degree

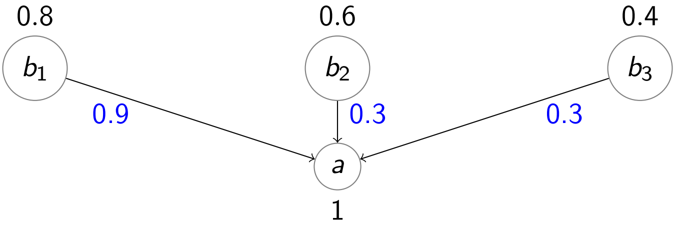
[a] My salary is quite high then I can rent the apartment.
[b_1] If the bank approves the loan to buy a car, I will not have enough money to rent the apartment.
[b_2] I may adopt a dog, however the owner forbids pets, thus I will not rent the apartment.
[b_3] I may adopt a cat, however the owner forbids pets, thus I will not rent the apartment.



Weighted Relations

For instance: relevance degree

[a] My salary is quite high then I can rent the apartment.
[b_1] If the bank approves the loan to buy a car, I will not have enough money to rent the apartment.
[b_2] I may adopt a dog, however the owner forbids pets, thus I will not rent the apartment.
[b_3] I may adopt a cat, however the owner forbids pets, thus I will not rent the apartment.



Similarity between Arguments

[b_1] If the bank approves the loan to buy a car, I will not have enough money to rent the apartment.
[b_2] I may adopt a dog, however the owner forbids pets, thus I will not rent the apartment.
[b_3] I may adopt a cat, however the owner forbids pets, thus I will not rent the apartment.

Similarity between arguments:

$$\mathbf{s}(b_1, b_2) = \mathbf{s}(b_1, b_3) = 0.2$$

$$\mathbf{s}(b_2, b_3) = 0.9$$

Argumentation Framework

Definition (AF)

An argumentation framework ($\mathtt{AF}$) is a tuple $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma, \mathbf{s} \rangle$, where

- $\mathcal{A} \subseteq_f \mathtt{Args}$ [a]
- $\mathbf{w} : \mathcal{A} \rightarrow [0, 1]$
- $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ (Attack relation)
- $\sigma : \mathcal{R} \rightarrow [0, 1]$
- $\mathbf{s} : \mathcal{A} \times \mathcal{A} \rightarrow [0, 1]$ (Similarity measure)

[a] $\mathtt{Args}$ denotes the universe of all possible arguments

Evaluation of Argument Strength

A degree (**strength**) of each argument

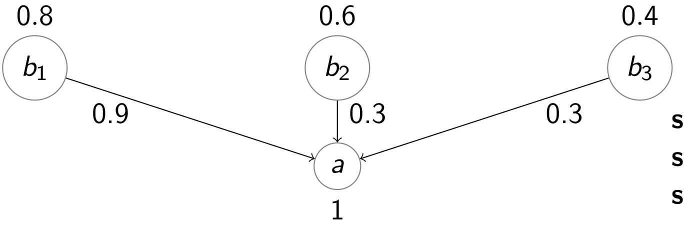
computes

Gradual Semantics

characterizes

Evaluation Method $= \langle \mathbf{f}, \mathbf{g}, \mathbf{h}, \mathbf{n} \rangle$

Evaluation Method $\mathcal{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{h}, \mathbf{n} \rangle$



$\mathbf{s}(b_1, b_2) = 0.2$
$\mathbf{s}(b_1, b_3) = 0.2$
$\mathbf{s}(b_2, b_3) = 0.9$

1. Assess the strenght of every attack (b_i, a),
   $\alpha_1 = \mathbf{h}(0.8, 0.9)$, $\alpha_2 = \mathbf{h}(0.6, 0.3)$, $\alpha_3 = \mathbf{h}(0.4, 0.3)$
2. Adjust the strenght of every attack w.r.t. similarity,
   $\beta = \mathbf{n}((\alpha_1, b_1), (\alpha_2, b_2), (\alpha_3, b_3)) = (\beta_1, \beta_2, \beta_3)$
3. Assess the strength of the group of attacks on a,
   $\delta = \mathbf{g}(\beta_1, \beta_2, \beta_3)$
4. Evaluate the impact of attacks on the initial weight of a,
   $\lambda = \mathbf{f}(1, \delta)$

Evaluation Method $\mathcal{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{h}, \mathbf{n} \rangle$

Definition (EM)

An *evaluation method* (EM) is a tuple $\mathcal{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{h}, \mathbf{n} \rangle$ such that:

- $\mathbf{f} : [0,1] \times \mathtt{Range}(\mathbf{g})$ [a] $\rightarrow [0,1]$,
- $\mathbf{g} : \bigcup_{k=0}^{+\infty} [0,1]^k \rightarrow [0,+\infty[$,
- $\mathbf{h} : [0,1] \times [0,1] \rightarrow [0,1]$,
- $\mathbf{n} : \bigcup_{k=0}^{+\infty} ([0,1] \times \mathtt{Args})^k \rightarrow [0,1]^k$.

[a] $\mathtt{Range}(\mathbf{g})$ denotes the co-domain of $\mathbf{g}$

Gradual Semantics

Definition (Gradual Semantics - $\mathcal{S}$)

$\mathcal{S}$ based on an evaluation method $\mathcal{M} = \langle \mathbf{f}, \mathbf{g}, \mathbf{h}, \mathbf{n} \rangle$ is a function assigning to every AF $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma, \mathbf{s} \rangle$, a weighting $\mathtt{Deg}_{\mathbf{G}}^{\mathcal{S}} : \mathcal{A} \rightarrow [0, 1]$ such that $\forall a \in \mathcal{A}$,

$$\mathtt{Deg}_{\mathbf{G}}^{\mathcal{S}}(a) = \mathbf{f}\left(\mathbf{w}(a), \mathbf{g}\left(\mathbf{n}\left((\mathbf{h}(\mathtt{Deg}_{\mathbf{G}}^{\mathcal{S}}(b_1), \sigma(b_1, a)), b_1), \cdots, (\mathbf{h}(\mathtt{Deg}_{\mathbf{G}}^{\mathcal{S}}(b_k), \sigma(b_k, a)), b_k)\right)\right)\right),$$

where $\{b_1, \cdots, b_k\} = \mathtt{Att}(a)$[a].

[a] $\mathtt{Att}(a)$ denotes the set of attackers of a

$\mathtt{Deg}_{\mathbf{G}}^{\mathcal{S}}(a)$ represents the strength of a

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Weighted h-Categorizer

Definition (Weighted h-Categorizer)

The function $\mathcal{S}_{\texttt{wh}}$ transforms any AF $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma, \mathbf{s} \equiv 0 \rangle$, into a weighting $\texttt{Deg}_{\mathbf{G}}^{\mathcal{S}_{\texttt{wh}}} : \mathcal{A} \rightarrow [0, 1]$ s.t. $\forall a \in \mathcal{A}$,

$$\texttt{Deg}_{\mathbf{G}}^{\mathcal{S}_{\texttt{wh}}}(a) = \begin{cases} \mathbf{w}(a) & \text{iff } \texttt{Att}(a) = \emptyset \\ \dfrac{\mathbf{w}(a)}{1 + \sum\limits_{b \in \texttt{Att(a)}} \texttt{Deg}_{\mathbf{G}}^{\mathcal{S}_{\texttt{wh}}}(b) \times \sigma(b,a)} & \text{else} \end{cases}$$

$\mathcal{S}_{\texttt{wh}}$ is based on $\mathcal{M} = \langle \mathbf{f}_{\texttt{frac}}, \mathbf{g}_{\texttt{sum}}, \mathbf{h}_{\texttt{prod}} \rangle$ such that:

$$\begin{cases} \mathbf{f}_{\texttt{frac}}(x_1, x_2) = \frac{x_1}{1+x_2} \\ \mathbf{g}_{\texttt{sum}}(x_1, \cdots, x_n) = \sum\limits_{i=1}^{n} x_i \\ \mathbf{h}_{\texttt{prod}}(x_1, x_2) = x_1 \times x_2 \end{cases}$$

Adjustment function $\mathbf{n}_{\mathtt{wh}}$

Definition ($\mathbf{n}_{\mathtt{wh}}$)

$Let\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma, \mathbf{s} \rangle$ be an $\mathtt{AF}$, $x_1, \cdots, x_k \in [0,1]$, and $b_1, \cdots, b_k \in \mathcal{A}$. We define $\mathbf{n}_{\mathtt{wh}}$ as follows:

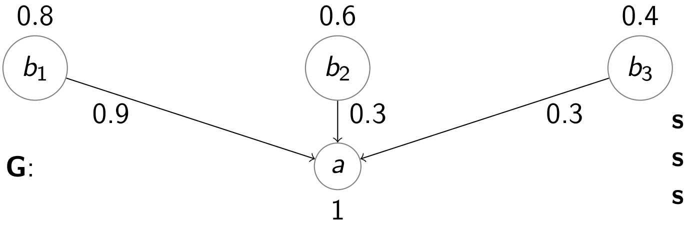
$$\mathbf{n}_{\mathtt{wh}}((x_1, b_1), \cdots, (x_k, b_k)) = (\mathtt{Deg}^{\mathcal{S}_{\mathtt{wh}}}_{\mathbf{G}'}(b_1), \cdots, \mathtt{Deg}^{\mathcal{S}_{\mathtt{wh}}}_{\mathbf{G}'}(b_k))$$

where $\mathbf{G}' = \langle \mathcal{A}', \mathbf{w}', \mathcal{R}', \sigma', \mathbf{s}' \rangle$, such that:

- $\mathcal{A}' = \{b_1, \cdots, b_k\}$,
- $\mathbf{w}'(b_1) = x_1, \cdots, \mathbf{w}'(b_k) = x_k$,
- $\mathcal{R}' = \{(b_1, b_2), \cdots, (b_1, b_k), \cdots, (b_k, b_1), \cdots, (b_k, b_{k-1})\}$,
- $\forall (b_i, b_j) \in \mathcal{R}'$, $\sigma'(b_i, b_j) = \mathbf{s}(b_i, b_j)$,
- $\mathbf{s}' \equiv 0$.

Example of $\mathbf{n}_{\mathtt{wh}}$

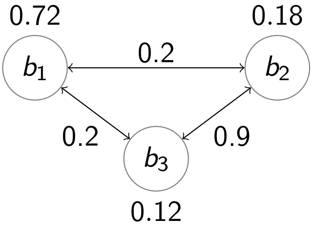
$\mathbf{G}$:



$\mathbf{s}(b_1, b_2) = 0.2$
$\mathbf{s}(b_1, b_3) = 0.2$
$\mathbf{s}(b_2, b_3) = 0.9$

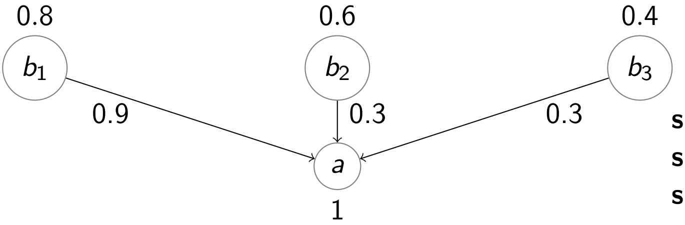
Assume that we use $\mathbf{h}_{\mathtt{prod}}(x_1, x_2) = x_1 \times x_2$

$\mathbf{G}'$:



$$\mathbf{n}_{\mathtt{wh}}((0.72, b_1), (0.18, b_2), (0.12, b_3))$$
$$= (\mathtt{Deg}^{\mathcal{S}_{\mathtt{wh}}}_{\mathbf{G}'}(b_1), \mathtt{Deg}^{\mathcal{S}_{\mathtt{wh}}}_{\mathbf{G}'}(b_2), \mathtt{Deg}^{\mathcal{S}_{\mathtt{wh}}}_{\mathbf{G}'}(b_3))$$
$$= (0.687, 0.147, 0.094)$$

Example of $\mathtt{EM}_{\mathtt{wh}} = \langle \mathbf{f}_{\mathtt{frac}}, \mathbf{g}_{\mathtt{sum}}, \mathbf{h}_{\mathtt{prod}}, \mathbf{n}_{\mathtt{wh}} \rangle$



$\mathbf{s}(b_1, b_2) = 0.2$
$\mathbf{s}(b_1, b_3) = 0.2$
$\mathbf{s}(b_2, b_3) = 0.9$

1. Strenght of every attack, $\alpha_1 = \mathbf{h}_{\mathtt{prod}}(0.8, 0.9) = 0.72$, $\alpha_2 = \mathbf{h}_{\mathtt{prod}}(0.6, 0.3) = 0.18$, $\alpha_3 = \mathbf{h}_{\mathtt{prod}}(0.4, 0.3) = 0.12$
2. Strenght of every attack w.r.t. similarity, $(\beta_1, \beta_2, \beta_3) = \mathbf{n}_{\mathtt{wh}}((\alpha_1, b_1), (\alpha_2, b_2), (\alpha_3, b_3)) = (0.687, 0.147, 0.094)$
3. Strength of the group of attacks on a, $\delta = \mathbf{g}_{\mathtt{sum}}(\beta_1, \beta_2, \beta_3) = 0.928$
4. Strength of a, $\lambda = \mathbf{f}_{\mathtt{frac}}(1, \delta) = 0.519$

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Some Properties on $\mathbf{n}_{\mathtt{wh}}$

$\mathbf{n}_{\mathtt{wh}}$ can be used by a gradual semantics

Theorem

There exists a unique semantics that is based on the evaluation method $\langle \mathbf{f}_{\mathtt{frac}}, \mathbf{g}_{\mathtt{sum}}, \mathbf{h}_{\mathtt{prod}}, \mathbf{n}_{\mathtt{wh}} \rangle$.

Some Properties of $\mathbf{n}_{\mathtt{wh}}$

When all the arguments are dissimilar, the adjustment function doesn't alter their initial values

Property (P1)

For any AF $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma, \mathbf{s} \rangle$, *for all* $a_1, \cdots, a_n \in \mathcal{A}$, *for all* $x_1, \cdots, x_n \in [0, 1]$, *if* $\forall i, j \in \{1, \cdots, n\}$, $i \neq j$, $\mathbf{s}(a_i, a_j) = 0$, *then*

$$\mathbf{n}_{\mathtt{wh}}((x_1, a_1), \cdots, (x_n, a_n)) = (x_1, \cdots, x_n).$$

$\mathbf{n}_{\mathtt{wh}}$ can only reduce the value of an argument

Property (P2)

For any AF $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma, \mathbf{s} \rangle$, *for all* $a_1, \cdots, a_n \in \mathcal{A}$, *for all* $x_1, \cdots, x_n \in [0, 1]$, *if* $\mathbf{n}_{\mathtt{wh}}((x_1, a_1), \cdots, (x_n, a_n)) = (x'_1, \cdots, x'_n)$, *then* $\forall i \in \{1, \cdots, n\}$, $x'_i \leq x_i$.

Some Properties of $\mathbf{n}_{\mathtt{wh}}$

If an argument is dissimilar to all other arguments and its initial value is 0, then it will not have any impact on the readjusted values of the other arguments

Property (P3)

For any AF $\mathbf{G} = \langle \mathcal{A}, \mathbf{w}, \mathcal{R}, \sigma, \mathbf{s} \rangle$*, for all* $a_1, \cdots, a_n, b \in \mathcal{A}$*, for all* $x_1, \cdots, x_n, y \in [0, 1]$*, if*

- $\forall i \in \{1, \cdots, n\}$, $\mathbf{s}(a_i, b) = 0$,
- $y = 0$,

then

$\mathbf{n}_{\mathtt{wh}}((x_1, a_1), \cdots, (x_n, a_n), (y, b)) = (\mathbf{n}_{\mathtt{wh}}((x_1, a_1), \cdots, (x_n, a_n)), 0).$

Outline

Readjusted Score - [Amgoud and al., KR 2018]

avg denotes the average operator

Definition (Readjusted Score $\mathbf{n}_{\mathtt{rs}}$)

Let $a_1, \cdots, a_k \in \mathtt{Args}$ and $x_1, \cdots, x_k \in [0, 1]$.
$\mathbf{n}_{\mathtt{rs}}((x_1, a_1), \cdots, (x_k, a_k)) =$

$$\left(\underset{x_i \in \{x_1, \cdots, x_k\} \setminus \{x_1\}}{\mathtt{avg}} \left(\frac{\mathtt{avg}(x_1, x_i) \times (2 - \mathbf{s}(a_1, a_i))}{2} \right), \cdots, \underset{x_i \in \{x_1, \cdots, x_k\} \setminus \{x_k\}}{\mathtt{avg}} \left(\frac{\mathtt{avg}(x_k, x_i) \times (2 - \mathbf{s}(a_k, a_i))}{2} \right) \right).$$

$\mathbf{n}_{\mathtt{rs}}() = ()$ and $\mathbf{n}_{\mathtt{rs}}((x_1, a_1)) = (x_1)$ if $k = 1$.

Properties on $\mathbf{n}_{\mathtt{rs}}$

Property

1. $\mathbf{n}_{\mathtt{rs}}$ *violates P1, P2 and P3*
2. *Let* $\mathbf{g}_{\mathrm{sum}}(x_1, \cdots, x_n) = \sum_{i=1}^{n} x_i$, $\exists a_1, \cdots, a_n, b \in \mathtt{Args}$ *and* $x_1, \cdots, x_n, y \in [0, 1]$ *such that:*
   - $\forall i \in \{1, \cdots, n\}$, $\mathbf{s}(a_i, b) = 0$,
   - $y = 0$,
   - $\mathbf{g}_{sum}(\mathbf{n}_{\mathtt{rs}}((x_1, a_1), \cdots, (x_n, a_n))) <$ $\mathbf{g}_{sum}(\mathbf{n}_{\mathtt{rs}}((x_1, a_1), \cdots, (x_n, a_n), (y, b)))$

Conclusions and Perspectives

Conclusions:

- Extending the notion of evaluation method by an adjustment function
- Proposing a novel adjustment function $\mathbf{n}_{\mathtt{wh}}$
- Investigating the properties of the existing functions

Perspectives:

- Axiomatisation of adjustment functions
- Investigating other adjustment functions

Reference

[Amgoud and al., KR 2018]: Amgoud L, Bonzon E, Delobelle J, Doder D, Konieczny S, Maudet N. Gradual Semantics Accounting for Similarity between Arguments. In: KR; 2018. p. 88–97.