Expressiveness of SETAFs and Support-Free ADFs under 3-valued Semantics

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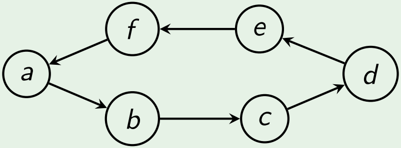
Introduction

- An *Argumentation Framework* (AF) is a pair $F = (N, R)$, [Dung, 1995]:
  - N: set of arguments,
  - $R \subseteq N \times N$: relation representing attacks between arguments.

Example

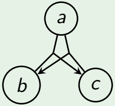


- Semantics: Methods used to clarify the acceptance of arguments
- Extension: set of jointly accepted arguments

Introduction

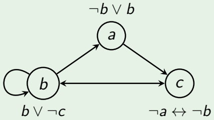
- Set Argument Frameworks (SETAFs) [Nielsen and Parsons, 2007]
  - Collective attack
  - Do not consider support among arguments

Example



- Abstract Dialectical Frameworks (ADFs) [Brewka and Woltran, 2010]
  - Unify several generalizations of AFs
  - Express relations between arguments beyond simple attack

Example



Motivation

Main Goal

Clarifying the expressiveness of SETAFs

- Two-valued signatures of SETAFs are more expressive than AFs [Dvořák et al., 2019]
- 3-valued labelling semantics of SETAFs are introduced in [Flouris and Bikakis, 2019]
- However, 3-valued signatures for SETAFs unexplored

Question

- Does the class of SETAFs embed in a subclass of ADFs, SFADFs?
- How is the characterization of SETAFs under 3-valued signatures?

Contribution

Main Contributions

- Introduce a subclass of ADFs
  - set abstract dialectical frameworks (SETADFs)
  - SETAFs and SETADFs coincide
- Comparing the expressiveness of SETADFs and SFADFs
- Characterising 3-valued signatures of SETAFs

Outline

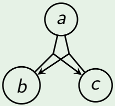
Background

Definition

A set argumentation framework (SETAF) is a pair (A, R) s.t.

- A is a finite set of arguments
- $R \subseteq (2^A \setminus \{\emptyset\}) \times A$ is the attack relation, $(B, a) \in R$

Example



3-valued labelling

- A 3-valued labelling: $\lambda : A \mapsto \{\mathtt{in}, \mathtt{out}, \mathtt{undec}\}$

Background

Conflict-free

Labelling λ is conflict-free if

- $\forall (S, a) \in R$ either $\lambda(a) \neq \texttt{in}$ or $\exists b \in S$ with $\lambda(b) \neq \texttt{in}$,
- $\forall a \in A$, if $\lambda(a) = \texttt{out}$ then $\exists (S, a) \in R$ s.t $\lambda(b) = \texttt{in}$ for all $b \in S$

Semantics of SETAFs

Given a SETAF $F = (A, R)$. Conflict-free labelling λ is

- $\lambda \in adm_{\mathcal{L}}$ if $\forall a \in A$ if $\lambda(a) = \texttt{in}$ then $\forall (S, a) \in R\ \exists b \in S$ s.t $\lambda(b) = \texttt{out}$;
- $\lambda \in comp_{\mathcal{L}}$ if $\forall a \in A$ (i) $\lambda(a) = \texttt{in}$ iff $\forall (S, a) \in R\ \exists b \in S$ s.t $\lambda(b) = \texttt{out}$, (ii) $\lambda(a) = \texttt{out}$ iff $\exists (S, a) \in R$ s.t $\lambda(b) = \texttt{in}\ \forall b \in S$;
- $\lambda \in grd_{\mathcal{L}}$ if it is complete and $\nexists \lambda'$ with $\lambda'_{\texttt{in}} \subset \lambda_{\texttt{in}}$ complete in F;
- $\lambda \in pref_{\mathcal{L}}$ if it is complete and $\nexists \lambda'$ with $\lambda'_{\texttt{in}} \supset \lambda_{\texttt{in}}$ complete in F;
- $\lambda \in stb_{\mathcal{L}}$ if $\lambda_{\texttt{undec}} = \emptyset$.

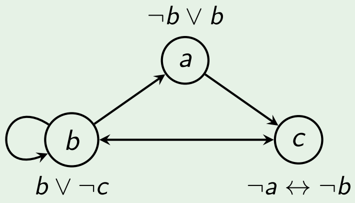
Background

Definition

An *abstract dialectical framework* (ADF) is a tuple $F = (A, L, C)$ where

- A is a finite set of nodes (arguments, statements)
- $L \subseteq A \times A$ is a set of links
- $C = \{\varphi_a\}_{a \in A}$ is a collection of propositional formulas (acceptance conditions)

Example



Background

3-valued interpretation

- A three-valued interpretation: $v : A \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$.

Semantics of ADFs

Given an ADF D. An interpretation v is

- $v \in adm(D)$ if $v \leq_i \Gamma_D(v)$
- $v \in pref(D)$ if v is $\leq_i$-maximal admissible
- $v \in comp(D)$ if $v = \Gamma_D(v)$
- v is $grd(D)$ if v is the $\leq_i$-least fixed point of $\Gamma_D(v)$
- $v \in mod(D)$ if v is a two-valued interpretation and $v = \Gamma_D(v)$
- $v \in stb(D)$ if V is a model of D and $v^t = w^t$, in which w is the grounded interpretation of $D^t = (v^t, L \cap (v^t \times v^t), \{\varphi_s[p/\bot \ : \ v(p) = f]\}_{s \in v^t})$
- $v \in cf(D)$ if for each $s \in S$; $v(s) = t$ implies φ_s^v is satisfiable and $v(s) = f$ implies φ_s^v is unsatisfiable

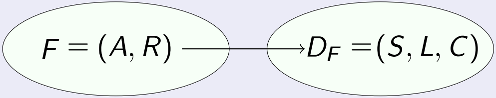
Embedding SETAFs in SETADFs

Definition

Given an ADF $D = (S, L, C)$

- support-free (SFADF): it contains only attacking links
- SETAF-like (SETADF): $\forall s \in S$: $\varphi_s : \bigwedge_{cl \in \mathcal{C}} \bigvee_{a \in cl} \neg a$

Lemma



SETAFs SETADFs

$$S = A$$
$$L = \{(a, b) \mid (B, b) \in R, a \in B\}$$
$$C = \{\varphi_a\} \text{ s.t.}$$
$$\varphi_a = \bigwedge_{(B,a) \in R} \bigvee_{a' \in B} \neg a'$$

Embedding SETAFs in SETADFs

Lemma

$D = (S, L, C) \longrightarrow F_D = (A, R)$

SETADFs SETAFs

$$A = S$$

$$\forall s \in S : \varphi_s = \bigwedge_{cl \in \mathcal{C}} \bigvee_{a \in cl} \neg a$$

we add $\{(cl, s) \mid cl \in \mathcal{C}\} \in R$

Theorem

Given a SETAF F and its associated SETADF D. For $\sigma \in \{cf, adm, comp, pref, grd, stb\}$, $\sigma_{\mathcal{L}}(F)$ and $\sigma(D)$ are in one-to-one correspondence.

SETAFs $F \longleftrightarrow D$ SETADFs

$$\sigma_{\mathcal{L}}(F) \longleftrightarrow \sigma(D)$$

For $\sigma \in \{cf, adm, comp, pref, grd, stb\}$

Realizability and Expressiveness

Definition [Dunne et al., 2015]

The signature of a formalism $\mathcal{C}$ under a semantics σ is defined as

$$\Sigma_{\mathcal{C}}^{\sigma} = \{\sigma(D) \mid D \in \mathcal{C}\}$$

Example

Given $\mathbb{V} = \{\{a \mapsto \mathbf{f}, b \mapsto \mathbf{u}, c \mapsto \mathbf{u}\}, \{a \mapsto \mathbf{f}, b \mapsto \mathbf{t}, c \mapsto \mathbf{f}\}, \{a \mapsto \mathbf{f}, b \mapsto \mathbf{f}, c \mapsto \mathbf{t}\}\}$.

- $\exists D \in \mathit{ADFs}$ s.t. $\mathbb{V} = \mathit{comp}(D)$?

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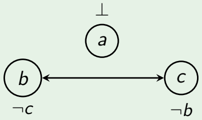
- $\exists D \in ADFs$ s.t. $\mathbb{V} = comp(D)$? Yes, $\mathbb{V} \in \Sigma_{ADF}^{comp}$



- $\exists D \in SETADFs$ s.t. $\mathbb{V} = comp(D)$?

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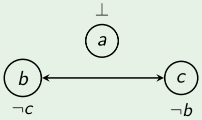


- $\exists D \in SETADFs$ s.t. $\mathbb{V} = comp(D)$? No, $\mathbb{V} \notin \Sigma_{SETADF}^{comp}$
- $\exists F \in SETAFs$ s.t. $\mathbb{V} = comp(F)$? No, $\mathbb{V} \notin \Sigma_{SETAF}^{comp}$

Relation between SETADFs and SFADFs

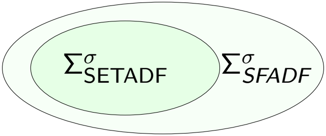
Example

Given $D = (\{a, b, c\}, \{\varphi_a : \neg c, \varphi_b : \neg a \wedge (\neg a \vee \neg c), \varphi_c : \neg a\})$.

- D is a SETADF,
- (c, b) is a redundant,
- D is not SFADF.

Lemma

For each SETADF D, $\exists$ an *equivalent* SETADF D' that is also a SFADF.



for $\sigma \in \{cf, adm, stb, mod, comp, pref, grd\}$

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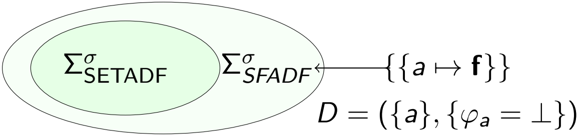

for $\sigma \in \{cf, adm, stb, mod, comp, pref, grd\}$

Relation between SETADFs and SFADFs

Lemma

Given a SFADF $D = (S, L, C)$. If $s \in S$ has a incoming link, then φ_s is in CNF containing only negative literals.

Example

Given $\mathbb{V} = \{\{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}\}\}$. For $\sigma \in \{stb, mod, comp, pref, grd\}$

- $\mathbb{V} \in \Sigma^{\sigma}_{SFADF}$?

Relation between SETADFs and SFADFs

Lemma

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Example

Given $\mathbb{V} = \{\{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}\}\}$. For $\sigma \in \{stb, mod, comp, pref, grd\}$

- $\mathbb{V} \in \Sigma^{\sigma}_{SFADF}$? Yes. $D = (\{a, b\}, \{\varphi_a : \top, \varphi_b : \bot\})$
- $\mathbb{V} \in \Sigma^{\sigma}_{SETADF}$?

Relation between SETADFs and SFADFs

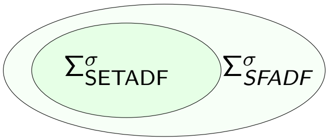
Lemma

Given a SFADF $D = (S, L, C)$. If $s \in S$ has a incoming link, then φ_s is in CNF containing only negative literals.

Example

Given $\mathbb{V} = \{\{a \mapsto \mathbf{t}, b \mapsto \mathbf{f}\}\}$. For $\sigma \in \{stb, mod, comp, pref, grd\}$

- $\mathbb{V} \in \Sigma^{\sigma}_{SFADF}$? Yes. $D = (\{a, b\}, \{\varphi_a : \top, \varphi_b : \bot\})$
- $\mathbb{V} \in \Sigma^{\sigma}_{SETADF}$? Yes. $D = (\{a, b\}, \{\varphi_a : \top, \varphi_b : \neg a\})$



$$\Delta_{\sigma} = \{\mathbb{V} \in \Sigma^{\sigma}_{\text{SFADF}} \mid \exists v \in \mathbb{V} \text{ s.t. } \forall a : v(a) \in \{\mathbf{f}, \mathbf{u}\} \wedge \exists a : v(a) = \mathbf{f}\}$$

Relation between SETADFs and SFADFs

Theorem

For $\sigma \in \{stb, mod, pref\}$ and $\mathbb{V} \in \Delta_\sigma$ ($\Delta_\sigma = \Sigma^{\sigma}_{\text{SFADF}} \setminus \Sigma^{\sigma}_{\text{SETADF}}$)

- $|\mathbb{V}| = 1$
- For $\sigma \in \{stb, mod\}$: $v = v^{\mathbf{f}}$

Relation between SETADFs and SFADFs

Theorem

For $\sigma \in \{stb, mod, pref\}$ and $\mathbb{V} \in \Delta_\sigma$ $(\Delta_\sigma = \Sigma^\sigma_{\text{SFADF}} \setminus \Sigma^\sigma_{\text{SETADF}})$

- $|\mathbb{V}| = 1$
- For $\sigma \in \{stb, mod\}$: $v = v^{\mathbf{f}}$

Example

Given SFADF $D = (\{a, b, c\}, \{\varphi_a = \bot, \varphi_b = \neg c, \varphi_c = \neg b\})$.

- $comp(D) = \{\{a \mapsto \mathbf{f}, b \mapsto \mathbf{u}, c \mapsto \mathbf{u}\}, \{a \mapsto \mathbf{f}, b \mapsto \mathbf{t}, c \mapsto \mathbf{f}\}, \{a \mapsto \mathbf{f}, b \mapsto \mathbf{f}, c \mapsto \mathbf{t}\}\}$,
- D is not *comp*-realizable in SETADF
- Since $comp(D) \subseteq adm(D) \subseteq cf(D)$, D is not σ-realizable in SETADFs, for $\sigma \in \{adm, cf\}$

3-valued Signatures of SETAFs

Proposition

The signature $\Sigma_{SETAF}^{pref_{\mathcal{L}}}$ is given by all non-empty sets $\mathbb{L}$ of labellings s.t.

1. all labellings $\lambda \in \mathbb{L}$ have the same domain $Args_{\mathbb{L}}$
2. If $\exists s$ s.t. $\lambda(s) = \mathtt{out}$, then $\lambda_{\mathtt{in}} \neq \emptyset$
3. $\forall \lambda_1, \lambda_2 \in \mathbb{L}$ if $\lambda_1 \neq \lambda_2$, then $\exists a$ s.t. $\lambda_1(a) = \mathtt{in}$ and $\lambda_2(a) = \mathtt{out}$

Proposition

The signature $\Sigma_{SETAF}^{stb_{\mathcal{L}}}$ is given by all sets $\mathbb{L}$ of labellings such that

1. $\mathbb{L} \in \Sigma_{SETAF}^{pref_{\mathcal{L}}}$
2. $\lambda(s) \neq \mathtt{undec}$ for all $\lambda \in \mathbb{L}$, $s \in Args_{\mathbb{L}}$

Summary and Future Work

Summary

- Each SETAF F is associated with a SETADF D, vice versa
- $\Sigma^{\sigma_{\mathcal{L}}}_{SETAF} \equiv \Sigma^{\sigma}_{SETADF}$
- SFADFs are more expressive than SETADFs and SETAFs
- Characterise $\Sigma^{\sigma_{\mathcal{L}}}_{SETAF}$, for $\sigma \in \{stb, pref, cf, grd\}$, under 3-valued signatures
- Indicate differences of $\Sigma^{pref_{\mathcal{L}}}_{SETAF}$ and $\Sigma^{stb_{\mathcal{L}}}_{SETAF}$ via 3-valued setting

Future Work

- Exact characterization of $\Sigma^{\sigma_{\mathcal{L}}}_{SETAF}$, for $\sigma \in \{adm, comp\}$
- Investigate whether the result improve the reasoning systems

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