Efficient Construction of Structured Argumentation Systems

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Let $T = (S, D)$ be a defeasible theory such that $S = \{r_1, r_2, r_3, r_4\}$ and $D = \{r_5, r_6, r_7\}$.

- $r_1$: “John has a prostate cancer” ($\rightarrow c$);
- $r_2$: “John is following a treatment for his cancer” ($\rightarrow t$);
- $r_3$: “John is a male patient” ($\rightarrow m$);
- $r_4$: “Studies show that there is no correlation between treating prostate cancer and bowel disorders” ($\rightarrow \neg r_7$);
- $r_5$: “John does not have abdominal pains” ($\Rightarrow a$);
- $r_6$: “If John does not have abdominal pain then he may not suffer from bowel disorder” ($a \Rightarrow \neg b$);
- $r_7$: “A patient with prostate cancer that is under treatment may suffer from bowel disorders” ($c, t \Rightarrow b$).
The ASPIC Framework

<table>
<thead>
<tr>
<th>Definition (Argument)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given $T = (S, D)$, an argument $A$ is $A_1, \ldots, A_n \leadsto \psi$, where ${A_1, \ldots A_n}$ is a minimal set of arguments s.t. $\exists r \in S \cup D$, where $r$ is applicable to ${\text{Conc}(A_1), \ldots, \text{Conc}(A_n)}$, $\leadsto = \text{Imp}(r)$ and $\psi = \text{Head}(r)$.</td>
</tr>
<tr>
<td>$\text{Conc}(A) = \psi$ and $\text{Sub}(A) = \text{Sub}(A_1) \cup \cdots \cup \text{Sub}(A_n) \cup {A}$ and $\text{TR}(A) = \text{Conc}(A_1), \ldots, \text{Conc}(A_n) \leadsto \psi$.</td>
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There are 7 arguments.

- $A_1 \Longrightarrow c$
- $A_3 \Longrightarrow m$
- $A_5 \Longrightarrow \neg r_7$
- $A_7 = A_4 \Rightarrow \neg b$
- $A_2 \Longrightarrow t$
- $A_4 \Rightarrow a$
- $A_6 = A_1, A_2 \Rightarrow b$
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There are 7 arguments.

$$A_7 = A_4 \Rightarrow \neg b.$$

$$Conc(A_7) = \neg b, \ TR(A_7) = a \Rightarrow \neg b \text{ and } Sub(A_7) = \{A_7, A_4\}.$$
We assume that we have a preference relation $\preceq$ on arguments.

**Definition (Defeat)**

$A$ defeats $B$ iff $B \preceq A$ and either:

1. **(Rebutting)** there exists $B' \in \text{Sub}(B)$ such that $\text{Conc}(B') = \neg \text{Conc}(A)$ and $\text{Imp}(\text{TR}(B'))$ is $\Rightarrow$; or
2. **(Undercutting)** $\text{Conc}(A) = \neg \text{Name}(\text{TR}(B))$
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- If $A_7 \prec A_6$ then $A_6 = A_1, A_2 \Rightarrow b$ defeats (rebutting) $A_7 = A_4 \Rightarrow \neg b$ but $A_7$ does not defeat $A_6$. 
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- If $A_6 \preceq A_5$ then $A_6 = A_1, A_2 \Rightarrow b$ is defeated by $A_5 = \rightarrow \neg r_7$ (undercutting)
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We get an argumentation system $(\text{AS}) \ A_{\text{AS}_T} = (A, \text{Def})$
The Idea Behind The Paper

We instantiate ASPIC on defeasible theories to reason with conflicting pieces of information.

We make the following observations:

- Most approaches generate the whole argumentation graph
- We have a huge number of arguments!
We instantiate ASPIC on defeasible theories to reason with conflicting pieces of information.

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- Most approaches generate the whole argumentation graph → Can we generate only a part of the AS?
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Contribution of the paper: The study of backward chaining mechanisms in ASPIC instantiations with an empirical evaluation demonstrating the impact of our approach.
Plan

1. Introduction
   - Structured Argumentation with ASPIC
   - Intuition

2. Backward Chaining for Argumentation
   - The Graph of Rule Interaction
   - AS for a literal
   - Defeasible Theory Filtration

3. Evaluation & Conclusion
   - Empirical Evaluation
   - Conclusion
The GRI expresses the interaction between the rules.

Definition (Graph of Rule Interaction)

The GRI of $\mathcal{T} = (S, D)$ is $\text{GRI}_\mathcal{T} = (\mathcal{N}, \mathcal{R}_s, \mathcal{R}_d)$, where:

1. $\mathcal{N} = S \cup D \cup \{\emptyset\}$;
2. $\mathcal{R}_s \subseteq 2^{\mathcal{N}} \times \mathcal{N}$ s.t. $\forall n_1 \in \mathcal{N}$ and $\forall N \subseteq \mathcal{N}$, $(N, n_1) \in \mathcal{R}_s$ iff $|N| = |\text{Body}(n_1)|$ and $\bigcup_{n \in N} \text{Head}(n) = \text{Body}(n_1)$.
3. $\mathcal{R}_d \subseteq \mathcal{N} \times \mathcal{N}$ s.t. $\forall n_1, n_2 \in \mathcal{N}$, $(n_1, n_2) \in \mathcal{R}_d$ iff at least one of the following conditions holds: (1) $\text{Head}(n_1) = \neg \text{Head}(n_2)$ and $\text{Imp}(n_2) \Rightarrow$ or (2) $\text{Head}(n_1) = \neg \text{Name}(n_2)$.

Note that $\text{Head}(\emptyset) = \text{Body}(\emptyset) = \text{Name}(\emptyset) = \emptyset$. 
The Graph of Rule Interaction

Figure – The corresponding graph of rule interaction
The sequence \( (\{\emptyset\}, \{\{r_1, r_2\}\}, \{\{r_7\}\}) \) is a support path to \( r_7 \).

**Figure** – A support path in the GRI
Characterising Rules

**Definition (Activated rule)**

A rule is activated iff there exists a support path to \( r \) in \( GRI_T \).

**Definition (Connected rule)**

\( n \) is connected to \( n' \) iff there exists \((n_1, \ldots, n_k)\) such that:

- for every \( 1 \leq i \leq k \), \( n_i \in \mathcal{N} \) and \( n_i \) is activated
- for every \( 1 \leq i \leq k - 1 \), it holds that either \((n_i, n_{i+1}) \in \mathcal{R}_d\) or there exists \( N \subseteq 2^\mathcal{N} \) s.t. \((N, n_{i+1}) \in \mathcal{R}_s\) with \( n_i \in N \).

**Definition (Potentially necessary rule)**

A rule \( r \) is potentially necessary for \( l \) iff \( \exists r' \in S \cup D \) s.t. \( \text{Head}(r') = l \) and \( r \) is connected to \( r' \).
Characterising Rules

$r_3$ is not potentially necessary for $b$ whereas $r_1, \ldots, r_6$ are.

Figure – Potentially necessary rules for $b$
Definition (AS for a literal)

\( \text{AS}_T^l = (A', \text{Def}') \) is the AS for \( l \) (w.r.t. \( \text{AS}_T = (A, \text{Def}) \)) iff \( A' \) is minimal (w.r.t. \( \subseteq \)) s.t. all the following are satisfied:

- \( A' \subseteq A \) and \( \{ a \in A \mid \text{Conc}(a) = l \} \subseteq A' \)
- If \( A \in A' \) and \( B \in A \) s.t. \( (B, A) \in \text{Def} \) then \( B \in A' \)
- If \( A \in A' \) then \( \text{Sub}(A) \subseteq A' \)
- \( \text{Def}' = \{(A, B) \in \text{Def} \mid A, B \in A'\} \).

| \( A_1 \rightarrow c \) | \( A_2 \rightarrow t \) |
| \( A_3 \rightarrow m \) | \( A_4 \implies a \) |
| \( A_5 \rightarrow \neg r_7 \) | \( A_6 = A_1, A_2 \Rightarrow b \) |
| \( A_7 = A_4 \Rightarrow \neg b \) |

Table – Arguments in the AS for \( b \)
The AS for \( l \) is useful to draw conclusions on \( l \)

Conclusions on other literals have to be taken carefully.

Links between the GRI and ASs:
- Sufficient condition to have less arguments in the AS for \( l \)
- Removing not activated rules will not change \( \text{AS}_T \)
- Removing not potentially necessary rules for \( l \) will not change the AS for \( l \) (\( \text{AS}_T^l \))
Defeasible Theory Filtration

We propose a framework inspired from the work of Yun et al.\(^3\) :

1. Compute the GRI of \(T\)
2. Compute and remove rules that are not potentially necessary for the literal.
3. Instantiate the AS for \(l\).

Please note that:

- The GRI only has to be computed once, stored in memory and reused for multiple queries.
- It can potentially reduce the time taken to answer a query.

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## Comparing BC and FC

<table>
<thead>
<tr>
<th>Theory</th>
<th>Forward</th>
<th>Backward</th>
<th>Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean time (s)</td>
<td># args.</td>
<td># defeats</td>
</tr>
<tr>
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<td></td>
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<td>n = 8, k = 3</td>
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<tr>
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<tr>
<td>levels(n)</td>
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<td>n = 7</td>
<td>11613.2</td>
<td>48896</td>
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</tr>
</tbody>
</table>

Table – Summary of the empirical evaluation
We provide a workflow for efficiently constructing structured AS using ASPIC on propositional logic:

- We introduce the Graph of Rule Interaction to study the behaviour of defeasible rules.
- We give theoretical properties for ASs for literals.
- We introduce a framework for filtering DTs.
- We empirically compare our new approaches and show promising results.
The Defeasible Theories

We consider the following DT from Maher et al.⁴:

- In \text{tree}(n,k), the rules form a \(k\)-branching tree of depth \(n\) where every literal occurs only once.
- In \text{level}(n), there is a cascade of \(n\) disputed conclusions, i.e. there are rules \(\Rightarrow p_i\) and \(p_{i+1} \Rightarrow \neg p_i\), for \(0 \leq i \leq n\).
- In \text{levels}(n), for odd \(i\), the latter rule has a superior strength when compared to even rules.
- In \text{teams}(n), every literal is disputed with two rules for \(p_i\) and two rules for \(\neg p_i\), and the rules for \(p_i\) are superior to the rules for \(\neg p_i\).

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