# Efficient Construction of Structured Argumentation Systems

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# The Underlying Language

Let $\mathcal{T} = (\mathcal{S}, \mathcal{D})$ be a defeasible theory such that $\mathcal{S} = \{r_1, r_2, r_3, r_4\}$ and $\mathcal{D} = \{r_5, r_6, r_7\}$.

- $r_1$ : "John has a prostate cancer" $(\rightarrow c)$ ;
- $r_2$ : "John is following a treatment for his cancer" $(\rightarrow t)$ ;
- $r_3$ : "John is a male patient" $(\rightarrow m)$ ;
- $r_4$ : "Studies show that there is no correlation between treating prostate cancer and bowel disorders" $(\rightarrow \neg r_7)$ ;
- $r_5$ : "John does not have abdominal pains" $(\Rightarrow a)$ ;
- $r_6$ : "If John does not have abdominal pain then he may not suffer from bowel disorder" $(a \Rightarrow \neg b)$ ;
- $r_7$ : "A patient with prostate cancer that is under treatment may suffer from bowel disorders" $(c, t \Rightarrow b)$.

# The ASPIC Framework

## Definition (Argument)

Given $\mathcal{T} = (\mathcal{S}, \mathcal{D})$, an argument $A$ is $A_1, \ldots, A_n \rightsquigarrow \psi$, where $\{A_1, \ldots A_n\}$ is a minimal set of arguments s.t. $\exists r \in \mathcal{S} \cup \mathcal{D}$, where $r$ is applicable to $\{Conc(A_1), \ldots, Conc(A_n)\}$, $\rightsquigarrow = Imp(r)$ and $\psi = Head(r)$.

$Conc(A) = \psi$ and $Sub(A) = Sub(A_1) \cup \cdots \cup Sub(A_n) \cup \{A\}$ and $TR(A) = Conc(A_1), \ldots, Conc(A_n) \rightsquigarrow \psi$.

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There are 7 arguments.

$$A_1 = \to c \qquad A_2 = \to t$$
$$A_3 = \to m \qquad A_4 = \Rightarrow a$$
$$A_5 = \to \neg r_7 \qquad A_6 = A_1, A_2 \Rightarrow b$$
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$$A_7 = A_4 \Rightarrow \neg b.$$

$Conc(A_7) = \neg b,\ TR(A_7) = a \Rightarrow \neg b$ and $Sub(A_7) = \{A_7, A_4\}$

# The ASPIC Framework

We assume that we have a preference relation $\preceq$ on arguments.

**Definition (Defeat)**

$A$ defeats $B$ iff $B \preceq A$ and either :

- (**Rebutting**) there exists $B' \in Sub(B)$ such that $Conc(B') = \neg Conc(A)$ and $Imp(TR(B'))$ is $\Rightarrow$ ; or
- (**Undercutting**) $Conc(A) = \neg Name(TR(B))$

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We get an argumentation system (AS) $\mathbb{AS}_{\mathcal{T}} = (\mathcal{A}, \text{Def})$

# The Idea Behind The Paper

We instantiate ASPIC on defeasible theories to reason with conflicting pieces of information.

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- We have a huge number of arguments !
  $\rightarrow$ How can we reduce the number of arguments ?

**Contribution of the paper :** The study of backward chaining mechanisms in ASPIC instantiations with an empirical evaluation demonstrating the impact of our approach.

# Plan

1. Introduction
   - Structured Argumentation with ASPIC
   - Intuition
2. Backward Chaining for Argumentation
   - The Graph of Rule Interaction
   - AS for a literal
   - Defeasible Theory Filtration
3. Evaluation & Conclusion
   - Empirical Evaluation
   - Conclusion

# The Graph of Rule Interaction

The GRI expresses the interaction between the rules.

**Definition (Graph of Rule Interaction)**

The GRI of $\mathcal{T} = (\mathcal{S}, \mathcal{D})$ is $GRI_{\mathcal{T}} = (\mathcal{N}, \mathcal{R}_s, \mathcal{R}_d)$, where :

1. $\mathcal{N} = \mathcal{S} \cup \mathcal{D} \cup \{\emptyset\}$ ;
2. $\mathcal{R}_s \subseteq 2^{\mathcal{N}} \times \mathcal{N}$ s.t. $\forall n_1 \in \mathcal{N}$ and $\forall N \subseteq \mathcal{N}, (N, n_1) \in \mathcal{R}_s$ iff $|N| = |Body(n_1)|$ and $\bigcup_{n \in N} Head(n) = Body(n_1)$.
3. $\mathcal{R}_d \subseteq \mathcal{N} \times \mathcal{N}$ s.t. $\forall n_1, n_2 \in \mathcal{N}, (n_1, n_2) \in \mathcal{R}_d$ iff at least one of the following conditions holds : (1) $Head(n_1) = \neg Head(n_2)$ and $Imp(n_2) = \implies$ or (2) $Head(n_1) = \neg Name(n_2)$.

Note that $Head(\emptyset) = Body(\emptyset) = Name(\emptyset) = \emptyset$.

# The Graph of Rule Interaction

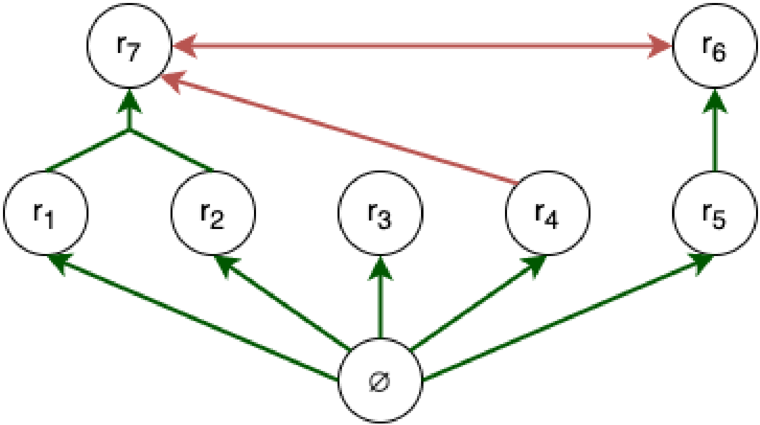


Figure – The corresponding graph of rule interaction

# Characterising Rules

The sequence $(\{\emptyset\}, \{\{r_1, r_2\}\}, \{\{r_7\}\})$ is a **support path** to $r_7$.

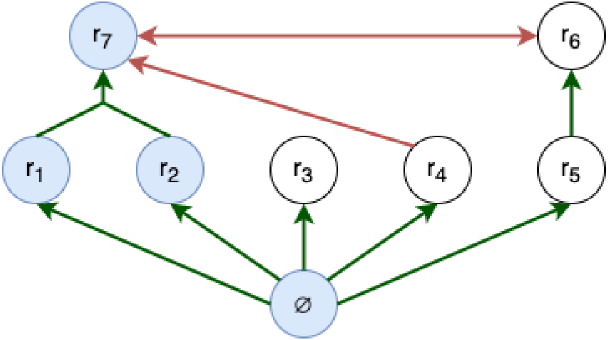


Figure – A support path in the GRI

# Characterising Rules

**Definition (Activated rule)**

A rule is activated iff there exists a support path to $r$ in $GRI_{\mathcal{T}}$.

**Definition (Connected rule)**

$n$ is connected to $n'$ iff there exists $(n_1, \ldots, n_k)$ such that :

- for every $1 \leq i \leq k$, $n_i \in \mathcal{N}$ and $n_i$ is activated
- for every $1 \leq i \leq k-1$, it holds that either $(n_i, n_{i+1}) \in \mathcal{R}_d$ or there exists $N \subseteq 2^{\mathcal{N}}$ s.t. $(N, n_{i+1}) \in \mathcal{R}_s$ with $n_i \in N$.

**Definition (Potentially necessary rule)**

A rule $r$ is potentially necessary for $l$ iff $\exists r' \in \mathcal{S} \cup \mathcal{D}$ s.t. $Head(r') = l$ and $r$ is connected to $r'$.

# Characterising Rules

$r_3$ is not potentially necessary for $b$ whereas $r_1, \ldots, r_6$ ared.

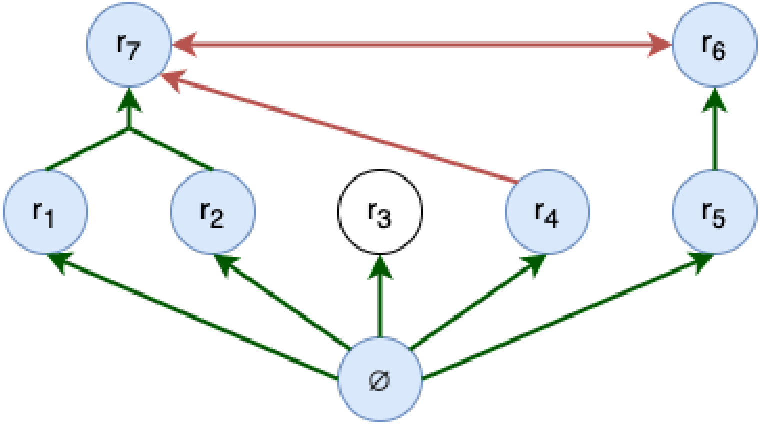


Figure – Potentially necessary rules for $b$

# Argumentation for a Literal

## Definition (AS for a literal)

$\mathbb{AS}^{l}_{\mathcal{T}} = (\mathcal{A}', \text{Def}')$ is the AS for $l$ (w.r.t. $\mathbb{AS}_{\mathcal{T}} = (\mathcal{A}, \text{Def})$) iff $\mathcal{A}'$ is minimal (w.r.t. $\subseteq$) s.t. all the following are satisfied :

- $\mathcal{A}' \subseteq \mathcal{A}$ and $\{a \in \mathcal{A} \mid Conc(a) = l\} \subseteq \mathcal{A}'$
- If $A \in \mathcal{A}'$ and $B \in \mathcal{A}$ s.t. $(B, A) \in \text{Def}$ then $B \in \mathcal{A}'$
- If $A \in \mathcal{A}'$ then $Sub(A) \subseteq \mathcal{A}'$
- $\text{Def}' = \{(A, B) \in \text{Def} \mid A, B \in \mathcal{A}'\}$.

| | |
|---|---|
| $A_1 = \to c$ | $A_2 = \to t$ |
| $A_3 = \to m$ | $A_4 = \Rightarrow a$ |
| $A_5 = \to \neg r_7$ | $A_6 = A_1, A_2 \Rightarrow b$ |
| $A_7 = A_4 \Rightarrow \neg b$ | |

Table – Arguments in the AS for $b$

# Theoretical Properties

- The AS for $l$ is useful to draw conclusions on $l$

  **Conclusions on other literals have to be taken carefully.**

- **Links between the GRI and ASs :**
  - Sufficient condition to have less arguments in the AS for $l$
  - Removing not activated rules will not change $\mathbb{AS}_{\mathcal{T}}$
  - Removing not potentially necessary rules for $l$ will not change the AS for $l$ ($\mathbb{AS}^{l}_{\mathcal{T}}$)

# Defeasible Theory Filtration

We propose a framework inspired from the work of Yun et al. [3] :

1. Compute the GRI of $\mathcal{T}$
2. Compute and remove rules that are not potentially necessary for the literal.
3. Instantiate the AS for $l$.

Please note that :

- The GRI only has to be computed once, stored in memory and reused for multiple queries.
- It can potentially reduce the time taken to answer a query.

3. Toward a More Efficient Generation of Structured Argumentation Graphs. Bruno Yun, Srdjan Vesic and Madalina Croitoru. COMMA (2018)

# Plan

# Comparing BC and FC

| Theory | Forward | | | Backward | | Filter | |
|---|---|---|---|---|---|---|---|
| | Mean time (s) | # args. | # defeats | Mean time (s) | Mean # arguments | % Filtration | Mean time (ms) |
| **tree(n,k)** | | | | | | | |
| $n=8, k=3$ | 5712.7 | 59049 | 0 | 19.6 | 15.8 | 99.93% | 484.6 |
| $n=9, k=3$ | Timeout | 196830 | 0 | 53.5 | 3.5 | 99.99% | 810.5 |
| **level(n)** | | | | | | | |
| $n=10$ | 0.1 | 19 | 26 | 0.03 | 10.8 | 43.16% | 41 |
| $n=1000$ | 6.7 | 1999 | 2996 | 830.5 | 716.6 | 64.15% | 132.60 |
| $n=5000$ | 181.7 | 9999 | 14996 | 3927.2 | 602.3 | 66.14% | 302.30 |
| $n=10000$ | 678.9 | 19999 | 29996 | Timeout | - | 57.88% | 490.40 |
| **levels(n)** | | | | | | | |
| $n=10$ | 0.1 | 19 | 18 | 0.04 | 14 | 26.32% | 46.5 |
| $n=1000$ | 6.7 | 1999 | 1998 | 1245.3 | 841.2 | 57.92% | 160.9 |
| $n=5000$ | 155.9 | 9999 | 9998 | 96542.02 | 3702 | 41.95% | 451.6 |
| $n=10000$ | 696.8 | 19999 | 19998 | 428.5 | 163 | 46.0% | 555 |
| **teams(n)** | | | | | | | |
| $n=3$ | 0.4 | 176 | 272 | 0.26 | 3.1 | 97.89% | 88 |
| $n=4$ | 1.6 | 736 | 1568 | 1.62 | 19.9 | 97.1% | 131.2 |
| $n=5$ | 26.8 | 3008 | 8256 | 5.28 | 23.8 | 99.14% | 198.3 |
| $n=6$ | 539.7 | 12160 | 254335 | 18.35 | 3.8 | 99.96% | 369.2 |
| $n=7$ | 11613.2 | 48896 | 1401159 | 84.07 | 14.1 | 99.97% | 866.5 |

Table – Summary of the empirical evaluation

# Conclusion

We provide a workflow for efficiently constructing structured AS using ASPIC on propositional logic :

- We introduce the Graph of Rule Interaction to study the behaviour of defeasible rules.
- We give theoretical properties for ASs for literals.
- We introduce a framework for filtering DTs.
- We empirically compare our new approaches and show promising results.

# The Defeasible Theories

We consider the following DT from Maher et al.[4] :

- In **tree(n,k)**, the rules form a $k$-branching tree of depth $n$ where every literal occurs only once.
- In **level(n)**, there is a cascade of $n$ disputed conclusions, i.e. there are rules $\Rightarrow p_i$ and $p_{i+1} \Rightarrow \neg p_i$, for $0 \leq i \leq n$.
- In **levels(n)**, for odd $i$, the latter rule has a superior strength when compared to even rules.
- In **teams(n)**, every literal is disputed with two rules for $p_i$ and two rules for $\neg p_i$, and the rules for $p_i$ are superior to the rules for $\neg p_i$.

4. Efficient Defeasible Reasoning Systems. M. J. Maher, A. Rock, G. Antoniou, D. Billington, and T. Miller. Int. J. on A.I. Tools (2001)