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A Discussion Game for the Grounded Semantics of Abstract Dialectical Frameworks

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Introduction

Dialectical methods and ADFs

- Dialectical method

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- Semantics of AFs are expressed by structural discussion [Vreeswijk and Prakken, 2000, Caminada, 2018]
- Methods used to interpret semantics of AFs cannot be reused in ADFs

Question

Are semantics of ADFs expressible in terms of structural discussion (discussion games)?

The first existing game for ADFs [Keshavarzi Zafarghandi et al., 2019] characterizes the preferred semantics.

Contribution

Main Contributions

- Defining a discussion game for grounded semantics of ADFs
  - *Grounded Discussion Game*
- Studying the soundness and completeness of the method
  - a claim of a defender has a winning strategy iff the grounded interpretation contains the claim

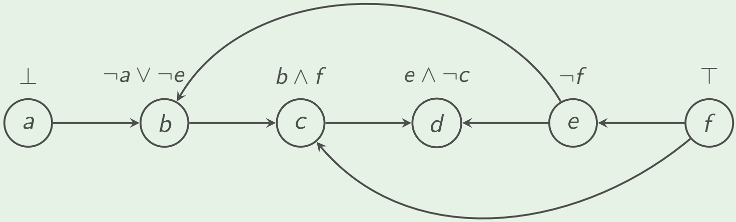
Background

Definition

An *abstract dialectical framework* (ADF) is a tuple $F = (A, L, C)$ where

- A is a finite set of nodes (arguments, statements)
- $L \subseteq A \times A$ is a set of links
- $C = \{\varphi_a\}_{a \in A}$ is a collection of propositional formulas (acceptance conditions)

Example



Background

Information ordering

Given an ADF $F = (A, L, C)$.

- A three-valued interpretation: $v : A \to \{\mathbf{t}, \mathbf{f}, \mathbf{u}\}$.
- $\leq_i$: $\mathbf{u} \leq_i \mathbf{t}$ and $\mathbf{u} \leq_i \mathbf{f}$.
- $v_i \leq_i v_j$ iff $\forall a \in A : v_i(a) \leq_i v_j(a)$.

Characteristic Operator

$$\Gamma_F(v) = v' \text{ s.t. } v'(a) = \begin{cases} \mathbf{t} & \text{if } \varphi_a^v \text{ is irrefutable (i.e., a tautology) },\\ \mathbf{f} & \text{if } \varphi_a^v \text{ is unsatisfiable,}\\ \mathbf{u} & \text{otherwise.} \end{cases}$$

$\varphi_a^v = \varphi_a[p/\top : v(p) = \mathbf{t}][p/\bot : v(p) = \mathbf{f}]$

Background

Semantics of ADFs

Given an ADF F. An interpretation v is

- $v \in adm(F)$ if $v \leq_i \Gamma_F(v)$
- v is $grd(F)$ if v is the $\leq_i$-least fixed point of $\Gamma_F(v)$

Acceptance and denial of arguments

Given an ADF F and an interpretation v

- $a \in A$ is *acceptable* w.r.t. v: φ_a^v is irrefutable
- $a \in A$ is *deniable* w.r.t. v: φ_a^v is unsatisfiable

Decision problem

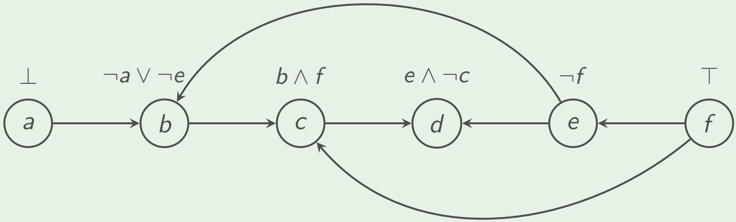
Given F an ADF, $a \in A$ and $\sigma \in \{adm, pref, comp, mod, \ldots\}$.

- a is *credulously acceptable (respectively, deniable)* under σ: if there exists a σ-interpretation that accepts (denies) a

Background

Example

$F = (\{a, b, c, d, e, f\}, \{\varphi_a : \bot, \varphi_b : \neg a \vee \neg e, \varphi_c : b \wedge f, \varphi_d : e \wedge \neg c, \varphi_e : \neg f, \varphi_f : \top\})$

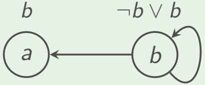


- v is $grd(F)$ if v is the $\leq_i$-least fixed point of $\Gamma_F(v)$
- $grd(F) = \{\{a \mapsto \mathbf{f}, b \mapsto \mathbf{t}, c \mapsto \mathbf{t}, d \mapsto \mathbf{f}, e \mapsto \mathbf{f}, f \mapsto \mathbf{t}\}\} = \{\mathbf{fttfft}\}$

Background

Example

$F = (\{a, b\}, \{\varphi_a : b, \varphi_b : \neg b \vee b\})$



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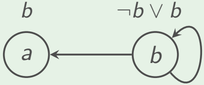

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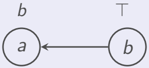
Theorem 4.2.13 of [Polberg, 2017]

- Any ADF F has an equivalent ADF F' without any redundant link.
- $F' = (\{a, b\}, \{\varphi_a : b, \varphi_b : \top\})$



Grounded discussion game

Structure of the game

- Two-player game: proponent, opponent
- Perfect information
- Socrates form of reasoning:
  - Proponent (P) presents a claim
  - Opponent (O) challenges the consequences of the claim
- P loses the game iff P cannot defeat challenges of O

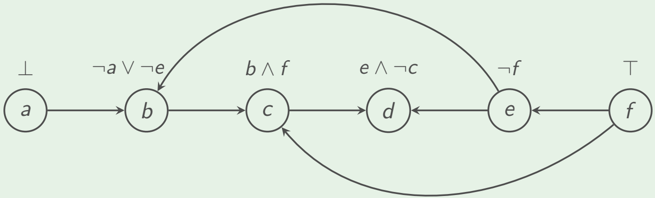
The main goal

Answering credulous (skeptical) decision problem of an ADF under grounded semantics *without* constructing the full grounded interpretation

Grounded discussion game

Example

- P claims: d is deniable, $g_0 = v_{\mathbf{u}}$



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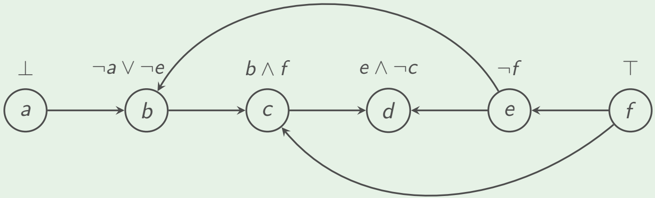
- O challenges: Is d an initial argument?



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- P claims: d is deniable, $g_0 = v_{\mathbf{u}}$
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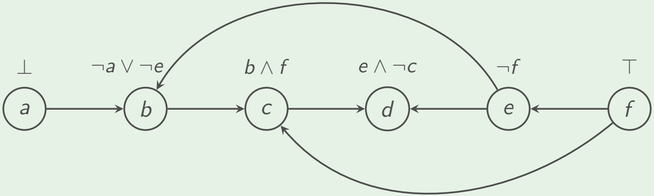
- P answers: No. $g_1 = g_0$



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- P claims: d is deniable, $g_0 = v_{\mathbf{u}}$
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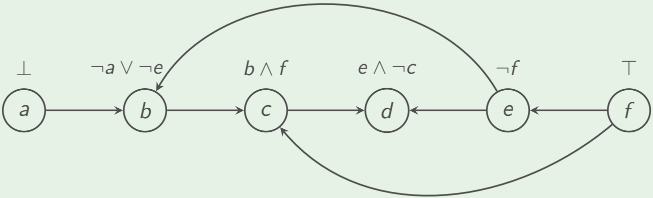
- O challenges: Is any of the ancestors of d an initial argument



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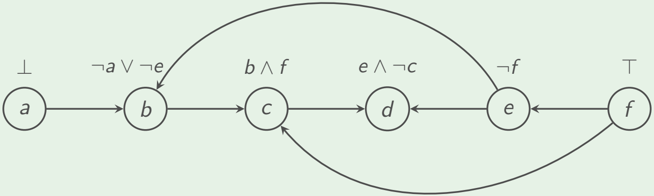
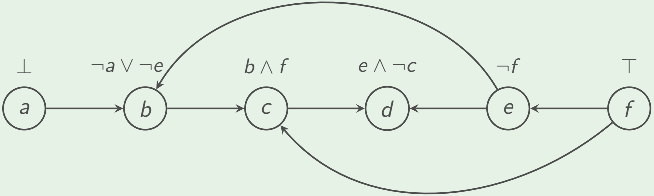
- P answers: $g_2 = g_1|^f_{\mathbf{t}} = \mathbf{uuuuut}$, $Ancestors(d, g_1) = \{d, e, c, f\}$



Grounded discussion game

Example

- P answers: $g_2 = g_1|^{f}_{\mathbf{t}} = \mathbf{uuuuut}$, $Ancestors(d, g_1) = \{d, e, c, f\}$
- O checks $g_1 <_i g_2$ and asks: extend g_2



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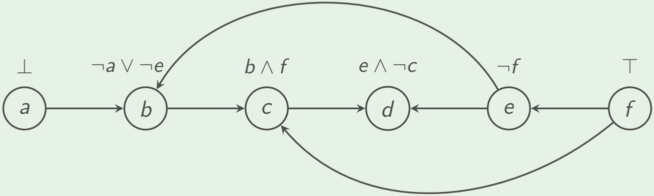
- O checks $g_1 <_i g_2$ and asks: extend g_2
- P evaluates $\varphi_c^{g_2} \equiv b \wedge \top \equiv b$, $\varphi_e^{g_2} \equiv \bot$: $g_3 = g_2|^e_{\mathbf{f}} = \mathbf{uuuuft}$



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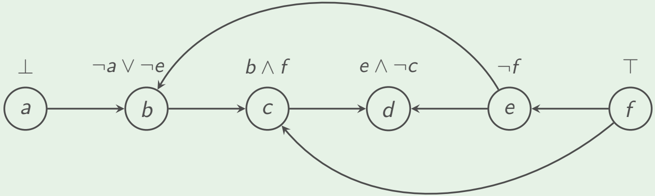
- O checks $g_2 <_i g_3$ and asks P to extend g_3



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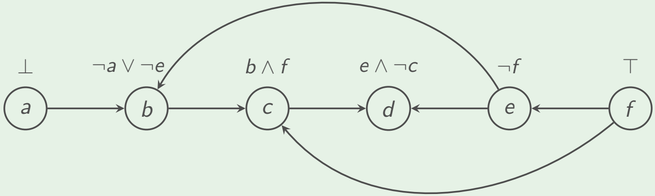
- O checks $g_2 <_i g_3$ and asks P to extend g_3
- P evaluates $\varphi_d^{g_3} \equiv \bot$: $g_4 = \mathbf{uuufft}$



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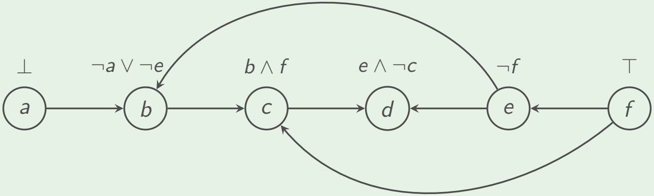
- O checks $g_2 <_i g_3$ and asks P to extend g_3
- P evaluates $\varphi_d^{g_3} \equiv \bot$: $g_4 = \mathbf{uuufft}$
- O checks $g_3 <_i g_4$: $d \mapsto \mathbf{f} \in g_4$



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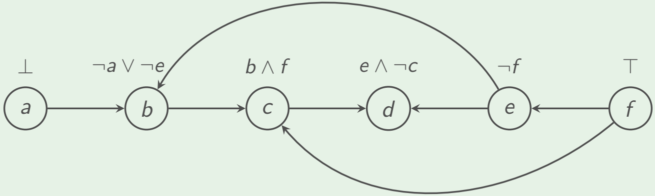
- O checks $g_2 <_i g_3$ and asks P to extend g_3
- P evaluates $\varphi_d^{g_3} \equiv \bot$: $g_4 = \mathbf{uuufft}$
- O checks $g_3 <_i g_4$: $d \mapsto \mathbf{f} \in g_4$
- The game stops: P wins



Grounded discussion game

- $IniClaim(a, x)$
- $Ini(a)$
- $CheckIni(a) : A \to \mathcal{V}$ $(g_1 = g_0|^{a}_{\mathbf{t}/\mathbf{f}} /\ g_1 = g_0)$
- $Check(g_{i-1}, g_i)$
  - If $g_{i-1} <_i g_i$ and g_i contains the initial claim: stop
  - If $g_{i-1} <_i g_i$: extend g_i (i.e. $Extend(g_i)$)
  - If $g_{i-1} \sim_i g_i$,
    - if $g_i = CheckIni(a)$ or $g_i = Eval(g_{i-1})$: O applies $IniAnc(a, g_{i-1})$,
    - if $g_i = NewIniAnc(a, g_{i-1})$: stop.
- $IniAnc(a, g)$
- $NewIniAnc(a, g) : A \times \mathcal{V} \to \mathcal{V}$ $(g|^{b}_{\varphi^{g}_{b}}$ s.t $b \in NewAnc(a, g))$
- $Ancestors(a, g) : A \times \mathcal{V} \to 2^A$
- $Extend(g)$
- $Eval(g) : \mathcal{V} \to \mathcal{V}$ $(g_{i+1} = g_i|^{b}_{\varphi^{g_i}_{b}})$

Grounded discussion game

Definition

Let $F = (A, R, C)$ be an ADF. A grounded discussion game for credulous acceptance (denial) of $a \in A$ is a sequence $[g_0, \ldots, g_n]$:

- $g_0 = v_{\mathbf{u}}$;
- $g_1 = CheckIni(a)$;
- for $0 \leq i < n$, $g_i \leq_i g_{i+1}$;
- for $1 < i < n$, if $g_{i-1} <_i g_i$: $g_{i+1} = Eval(g_i)$;
- for $0 < i < n$, if $g_{i-1} \sim g_i$: $g_{i+1} = NewIniAnc(a, g_i)$;
- g_n contains either
  - the initial claim,
  - or the negation of the initial claim, or $g_n = NewIniAnc(a, g_{n-1})$ and $g_{n-1} \sim g_n$.

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- g_n contains either
  - the initial claim,
    P wins the game
  - or the negation of the initial claim,
    or $g_n = NewIniAnc(a, g_{n-1})$ and $g_{n-1} \sim g_n$.
    O wins the game

Grounded discussion game

Lemma

Let F be an ADF without any redundant link, that does not have any initial argument. Then $grd(F) = v_{\mathbf{u}}$.

Grounded discussion game

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Let F be an ADF without any redundant link, that does not have any initial argument. Then $grd(F) = v_{\mathbf{u}}$.

Example

Let $F = (\{a, b\}, \{\varphi_a : b \vee \neg b, \varphi_b : b\})$. $grd(F) = \{a \mapsto \mathbf{t}, b \mapsto \mathbf{u}\}$.

Proposition

If $a \mapsto \mathbf{t}/\mathbf{f} \in grd(F)$, then either is an initial argument or has at least one initial ancestor.

Grounded discussion game

Lemma

Let F be an ADF without any redundant link, that does not have any initial argument. Then $grd(F) = v_{\mathbf{u}}$.

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Grounded discussion game (Completeness, Soundness)

Completeness

Let F be a given ADF without any redundant links. If a is acceptable (deniable) in the grounded interpretation of F, then there is a grounded discussion game for the initial claim of accepting (denying) of a.

Soundness

Let F be a given ADF. If there is a grounded discussion game for an initial claim of P in which P wins, then $grd(F)$ satisfies the initial claim of P.

Summary and Future Work

Summary

- We introduced the grounded discussion game.
- P tries to show that the initial claim can be in an extension of the trivial interpretation.
- O tries to challenge P.
- The method is sound and complete.
- The current method works locally over the ancestors of the argument in question.
- Even in the worst case, the method does not coincide with the least-fixed-point algorithm of grounded interpretation.

Future Work

- Does the method improve the reasoning system?
- Does the game work for infinite ADFs?

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