Continuum Argumentation Frameworks from Cooperative Game Theory

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Contents

Motivations

- Dung's 1995 seminal paper argues that abstract argumentation can be applied to problems of social relevance [4, Section 3].
- He initiated a correspondence between cooperative games and abstract argumentation.
- We have taken this correspondence further in two ways:
  - Correspondence of all four of Dung's extensions with solution concepts of cooperative games [13].
  - Investigate when / whether various argumentation framework (AF) properties defined by Dung hold for cooperative games.
- This is desirable because we want to explore how ideas from argumentation can be useful to cooperative game theory.
- Will assume all of you know abstract argumentation!

Cooperative Game Theory

- **Game theory:** The mathematical modelling of strategic interaction between rational decision makers (e.g. [8])
- **Cooperative game theory:** Game theory that assumes it is possible for players to make binding agreements (e.g. [3])
- Binding agreements incentivise rational players to cooperate to earn more payoff.

Agents and Coalition Values

- Let $N := \{1, 2, 3, \ldots, m\}$, for $m \in \mathbb{N}^+$, be a set of **agents**.
- A **coalition** is a subset of N.
- The **grand coalition** is N itself.
- A **valuation function** is $v : \mathcal{P}(N) \to \mathbb{R}$, where $v(\varnothing) = 0$.
- A **coalition game (in normal form)** is $\langle N, v \rangle$.
- Standard assumptions:
  - v is **non-negative**: $(\forall C \subseteq N)\, v(C) \geq 0$.
  - v is **superadditive**: $(\forall C, C' \subseteq N)\, [C \cap C' = \varnothing \Rightarrow v(C \cup C') \geq v(C) + v(C')]$.
  - v is **essential**: $\sum_{k=1}^{m} v(\{k\}) < v(N)$

Example

- Let $m = 3$, $N = \{1, 2, 3\}$
  1. $1 =$ David
  2. $2 =$ Josh
  3. $3 =$ Peter
- David, Josh or Peter earn no payoff if they work alone: $v(\{1\}) = v(\{2\}) = v(\{3\}) = 0$.
- If any two of them work together they earn 10 units of payoff, so $v(C) = 10$ if $|C| = 2$.
- If all three work together they earn 20 units of payoff, so $v(N) = 20$.
- This is non-negative, super-additive and essential.

Payoffs

- It is standard (e.g. [3, 12]) to assume that all m agents work together by forming the grand coalition, N
- Collectively, they receive $v(N)$ payoff.
- How should we distribute $v(N)$ back to the m players?
- Formalise payoff as $\mathbf{x} := (x_1, x_2, \ldots, x_m) \in \left(\mathbb{R}_0^+\right)^m$ such that agent 1 gets x_1... etc.
- **Transferable Utility:** we assume that $v(N)$ can be distributed *in any way* amongst the m players.
- e.g. $m = 3$, $v(N) = 20$, can have

$$\mathbf{x}_1 := (10, 10, 0) \text{ or } \mathbf{x}_2 := \left(\pi, e, \sum_{k=1}^{\infty} \frac{1}{k^3} \approx 1.202\right) \tag{1}$$

as payoffs.

Imputations and Domination

- We want to distribute $\mathbf{x}$ to the m players such that no one wants to **defect** from the grand coalition.
- Given $\langle N, v \rangle$, define the set of **imputations** IMP to be

$$\left\{ \mathbf{x} \in \left(\mathbb{R}_0^+\right)^m \middle| (\forall k \in N)\, x_k \geq v\left(\{k\}\right), \sum_{k=1}^{m} x_k = v(N) \right\}. \quad (2)$$

- For $\mathbf{x}, \mathbf{y} \in IMP$, we say $\mathbf{x}$ **dominates** $\mathbf{y}$, denoted $\mathbf{x} \to \mathbf{y}$, iff

$$(\exists C \subseteq N) \left[C \neq \varnothing, (\forall k \in C)\, x_k > y_k, \sum_{k \in C} x_k \leq v\left(C\right) \right]. \quad (3)$$

- $\langle IMP, \to \rangle$ is a directed graph, where if $m > 1$, $|IMP| = |\mathbb{R}|$.

Example

- David, Josh, Peter
- Each earns nothing if they work alone.
- Any two of them working together can collectively earn 10.
- Everyone working together can collectively earn 20.
- $(12, 4, 4) \rightarrow_{\{2,3\}} (20, 0, 0)$ because Josh and Peter can defect and form their own coalition, earning 10 and each can afford to take 4.
- $(10, 5, 5) \rightarrow_{\{2,3\}} (12, 4, 4)$ as well.

Solution Concepts Correspond to Argument Extensions

- Which payoff distributions (imputations) are "sensible" given $\langle N, v \rangle$?
- **Von Neumann-Morgenstern stable sets:** the stable extensions of $\langle IMP, \rightarrow \rangle$ when viewing $\langle IMP, \rightarrow \rangle$ as an abstract AF [3, 4, 12].
- But these may not exist [6, 7]. Gillies suggested the **core** [5], which corresponds to the set of unattacked arguments [4].
- The core may be empty [2, 10]. Roth suggested the **subsolutions** and the **supercore** [9], which always exist (iff AC, [11]). These correspond, respectively, to the complete and grounded extensions [13].
- (See [13] and the paper for examples of solution concepts.)

Properties of AFs for these Games

- We have further shown that $\langle IMP, \rightarrow \rangle$ for such games are *never* [4]:
  - **finitary:** all nodes have finite in-degree.
  - **well-founded:** no ω-"backwards" chains of attacks.
  - **uncontroversial:** no two arguments have both an odd and even-length path from one to the other.
  - **limited controversial:** no ω-"backwards" chains of controversies.
- Therefore, there is no straightforward way of reducing the multiplicity of solution concepts.
- See paper for proofs, which use straightforward ideas from set theory, analysis and topology.

Conclusions

- From a transferable-utility m-player game we can construct an uncountably infinite AF [4, 9], when $m > 1$.
- The four Dung extensions of this AF correspond exactly to the solution concepts of the game [4, 13].
- Such AFs do not obey various desirable properties of AFs that reduce the multiplicity of solutions (this paper).
- This is a "natural" example of an infinite AF (see, e.g. [1]).

Future Work

- Investigate **coherence** and **relative-groundedness** for such continuum AFs.
- From argumentation to cooperative game theory: what about non-Dung semantics such as **semi-stable**, **ideal**, **eager**... etc.?
- From cooperative game theory to argumentation: what about non-defection-based solution concepts such as the **Shapley value**, the **nucleolus**... etc.?
- Non-transferable utility...?
- Hope that this can encourage more interactions between argumentation and cooperative game theory.

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Thank you! Questions?

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